SINGULARITIES OF THE BREAKUP OF VISCOUS
LIQUID DROPLETS IN SHOCK WAVES

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The influence of the viscosity of the liquid on the breakup of droplets behind a shock front
has been investigated experimentally.

The addition of liquid-metal components to a fuel enhances the energy characteristics of the combustion processes [1]. Liquid components of this kind have from 10^2 to 10^3 times the viscosity of ordinary liquids. The increased viscosity may require a modification in the design and characteristics of the fuel-injection system. Moreover, the atomization of a viscous liquid is set apart by a number of highly specific attributes. The first consideration is felt in an increase of the friction associated with flow of the liquid, and the second has the effect of magnifying the damping influence of the liquid viscosity on the evolution of wave effects at its surface.

The laws governing the atomization of very viscous liquids has not been adequately studied. Very viscous liquids are regarded as those for which the Laplace number $L = \rho_f \sigma d_1 \mu^{f-2} \approx 1$. A growth of the Weber number $W* = 0.5 \rho u^2 d_1 \sigma^{-1}$ with the viscosity has been asserted in [2]. Hinze [3] has arrived at a theoretically similar conclusion in qualitative terms, but without determining the form of the function $W* = f(L)$. Sufficient data are also lacking on the dynamics of droplet breakup under conditions for which $L \approx 1$. Only in [4] is there mention of the retarding action of the viscosity in the breakup of droplets. A theoretical relation has been given in [5] for determining the droplet breakup time of a viscous liquid: $t = 32 \mu_f (\rho u^2)^{-1}$. The values of t calculated according to this relation exhibit large discrepancies with the breakup times determined experimentally in [4].

1. Experimental Procedure. The simplest way to determine the laws of droplet breakup is the widely used shock wave experimental procedure [6]. According to that procedure droplets of the investigated liquid are injected into the low-pressure chamber of a shock tube, where they are then quickly exposed to a gas flow behind a shock wave of known intensity. The drop sizes and evolution of the breakup process are recorded photographically by means of strobe lights. The shock wave parameters are measured with piezoelectric pressure pickups.

In our experiments we studied the breakup of drops of water and aqueous solutions of glycerin with different concentrations. The values of the surface tension σ, N/m, density of the liquid ρ_f, kg/m³, and viscosity μ_f, N·sec/m² for the investigated liquids are summarized in Table 1. As the latter indicates,

TABLE 1

Liquid	Parameter		
	$\sigma \cdot 10^3$	$\rho_f \cdot 10^{-3}$	$\mu_f \cdot 10^3$
H_2O	73	1	1
42% H_2O	66,4	1,14	9,15
20% H_2O	66,6	1,2	51,9
80% glycerin			
Glycerin	64	1,26	1480

Fig. 1. Critical Weber number versus Laplace number.

liquids were chosen for which $\sigma \approx$ const and $\rho_f \approx$ const. The viscosities of the liquids differ by a factor of more than 10^3. The droplet sizes were measured in the range d_1 = 0.3 to 1.5 mm, and the Mach numbers of the shock waves lay in the interval M = 1.05 to 1.15. The inital pressure and temperature of the nitrogen in the low-pressure chamber were p = 1 atm and T = 293°K.

2. Experimental Results. Figure 1 shows the variation of the critical Weber number W_μ^* for viscous liquid droplets. For kerosene and water as liquids having negligible viscosity the critical Weber number is W* $\approx$ 5. The Laplace number for the investigated range of droplet sizes for such liquids was L $\approx 10^4 \gg 1$. The graph indicates that with a tenfold increase in the liquid viscosity (in which case the Laplace number changes by a factor of 10^2) a variation of the critical Weber number is not noted. A further increase in the viscosity, however, causes an appreciable growth of W_μ^*. Thus, for L $\approx 3.6 \cdot 10^{-2}$ in the case of glycerin $W_\mu^* \approx 25$, i.e., is five times the value for kerosene. Besides the values of W_μ^* obtained in our experiments (points 1), the figure also gives data taken from [2,3] (points 2 and 3, respectively). Consequently, the breakup of viscous liquid droplets, all other conditions being equal, requires a large dynamic head of the gas, because $0.5\rho u^2 = \sigma d_1^{-1} W_\mu^*$. The critical value of the Weber number for droplets of a particular liquid can be determined from the relation

$$W_\mu^* \approx 5(1 + 1.5L^{-0.37}).$$

This relation is valid in the interval of Laplace numbers $10^{-2} < L < 10^5$ and for drop diameters of 0.1 to 2 mm at surface tensions $15 < \sigma < 400$ N/m.

It follows from the dependence $W_\mu^* = f(L)$ that the critical Weber number depends in general also on the droplet diameter at constant $\sigma_f \rho_f$ and μ_f, a decrease of the droplet diameter causing an increase of W_μ^*. An analysis of the experimental data shows that a 1.5-fold increase of the Weber number W_μ^* can be observed for low-viscosity liquids such as kerosene, water, and liquid nitrogen in the range of droplet diameters from 10 to 100 μ. The tendency toward an increase in the critical Weber number for small droplets of more viscous liquids is even more appreciable. The dependence of W_μ^* on the droplet size is plotted in Fig. 2 for three liquids with viscosities of 10^{-3}, $4.8 \cdot 10^{-3}$, and $9.7 \cdot 10^{-2}$ kg/m·sec (curves 1, 2, and 3, respectively) and is compared with the experimental data of [2] (points 1, 2, and 3). A variation of the droplet diameter has no effect on the critical Weber number only for drops with diameters $d_0 < 800 \mu_t^2$ $(\rho_f \sigma)^{-1}$. For inviscid liquids the values of d_0 amount to less than 10 μ. For liquids with a high viscosity the influence of the droplet diameter is already felt for $d_0 \approx 0.1$ mm.

We now examine the variation with time t of the transverse dimensions d of disintegrating droplets of liquids having different viscosities (Fig. 3). We analyze the variation of d in coordinates d* = dd_1^{-1} and t* = $t\tau^{-1}$, where $\tau = d_1 \rho_f^{0.5} (\rho u^2)^{-0.5}$ and u, ρ are the velocity and density of the gas behind the shock wave. Curves 1, 2, and 3 represent liquid droplets with a diameter d_1 = 1 mm and Laplace numbers 10^4,
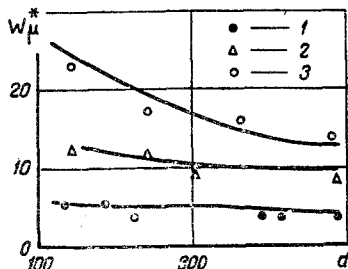

Fig. 2

Fig. 3

Fig. 4

Fig. 2. Critical Weber number versus droplet diameter d (in μ). 1) $\mu_f = 10^{-3}$; 2) $4.8 \cdot 10^{-3}$; 3) $9.7 \cdot 10^{-2}$ kg/m·sec.

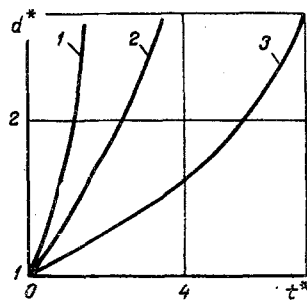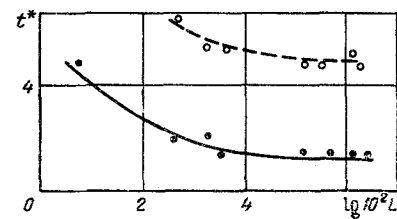Fig. 3. Distortion curves for liquid droplets with different viscosities. 1) L = 10^4; 2) 31; 3) $3.6 \cdot 10^{-2}$.

Fig. 4. Time of attainment of maximum distortion and time of complete droplet breakup versus Laplace number.

31, and $3.6 \cdot 10^{-2}$. It is readily observed from a comparison of the curves d = d(t) how the droplet distortion process is retarded with an increase in the viscosity. Thus, whereas for $L \gg 1$ the critical distortion stage d* ≈ 3 sets in at t* ≈ 1.5, for L < 1 that time increases to t* = 6 or 7. The breakup process is stretched out over a time up to t* ≈ 8 to 10 (dashed curve in Fig. 4), by contrast with t* ≈ 4 to 5 for droplets with $L \gg 1$. Simultaneously we observe an appreciable increase in the induction time for the initiation of the detachment of a thin liquid film from the droplet (solid curve in Fig. 4). The increase in the induction time $t^*_{i\mu}$ and total-breakup time t^*_b with increasing viscosity is described by the expressions

$$t^*_{i\mu} \approx 1.4 \, (1 + 1.5 L^{-0.37}),$$

$$t^*_b \approx 4.5 \, (1 + 1.2 L^{-0.37}).$$

These relations given above for the determination of the breakup time for water droplets, $t = 32 \mu_f (\rho u^2)^{-1}$, is completely inapplicable, because it fails in general to reflect the dependence of that time on the droplet diameter and yields an experimentally inconsistent picture of the variation of the breakup time with the viscosity of the liquid. The detachment of a surface film of liquid from droplets of viscous liquids is observed for a Weber number $W^{**} \approx$ (4 to 5)W^*_μ, whereas for $L \gg 1$ we find $W^{**} \approx 2W^*$.

The established retardation of the droplet distortion and breakup processes for viscous liquids implies that after the injection of such droplets into a particular volume they will penetrate deeper into the surrounding space. Therefore, to attain equal degrees of atomization for the viscous components of a fuel it is required to increase the dimensions of the combustion chamber. With the injection of viscous liquid jets into an entraining gas flow the depth of penetration of the jets into the latter should be observed to increase.

NOTATION

ρ_f	is the density of the liquid, kg/m^3;
μ_f	is the viscosity of the liquid, N·sec/m^2 or kg/m·sec;
d	is the droplet diameter, m;
σ	is the surface tension, N/m;
ρ	is the density of the gas, kg/m^3;
u	is the velocity of the gas, m/sec;
t	is the time, sec;
$L = \rho_f \sigma d / \mu_f^2$	is the Laplace number;
$W = 0.5 \rho u^2 d / \sigma$	is the Weber number;
M	is the Mach number;
T	is the temperature of the gas, °K;
p	is the pressure, atm.

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